

SOLUTIONS TO PRACTICE EXAM 1 EXTRAS, MATH 10560

1. Find the integral

$$\int \frac{3x+1}{x^3+x^2} dx.$$

**Solution:** Use partial fraction decomposition

$$\frac{3x+1}{x^3+x^2} = \frac{3x+1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} = \frac{Ax(x+1) + B(x+1) + Cx^2}{x^2(x+1)}.$$

Therefore

$$3x+1 = (A+C)x^2 + (A+B)x + B.$$

It follows that

$$\begin{aligned} A+C &= 0, & A+B &= 3, & B &= 1, \\ A &= 2, & B &= 1, & C &= -2, \end{aligned}$$

and

$$\int \frac{3x+1}{x^3+x^2} dx = \int \left( \frac{2}{x} + \frac{1}{x^2} - \frac{2}{x+1} \right) dx = 2 \ln|x| - \frac{1}{x} - 2 \ln|x+1| + C.$$

2. Calculate the integral

$$\int \frac{dx}{x + \sqrt[3]{x}}.$$

**Solution:** Make substitution  $u = x^{1/3}$ . Then  $u^3 = x$  and with  $dx = 3u^2 du$

$$\int \frac{dx}{x + \sqrt[3]{x}} = \int \frac{3u^2 du}{u^3 + u} = \int \frac{3u du}{u^2 + 1} = \frac{3}{2} \ln(u^2 + 1) + C = \frac{3}{2} \ln(x^{2/3} + 1) + C.$$